Performance of Precoded Integer-Forcing for Closed-Loop MIMO Multicast

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IEEE ITW 2014, November 4th

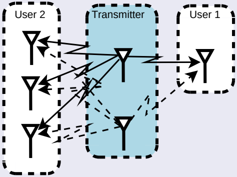
Introduction

- The Multiple-Input Multiple-Output (MIMO) Gaussian channel has been the focus of extensive research efforts.
- The MIMO Gaussian broadcast channel has also been widely studied for over a decade now.
- Private Message vs. Common Message
  - Capacity is known for both cases ✓
  - Practical scheme
    - Private Message ✓ (DPC)
    - Common Message ?

    $\Longrightarrow$ **Topic of this talk**

Challenges for Practical MIMO Multicast

- In contrast to the single-user case, the number of data streams, constellation size and other parameters cannot be tailored to a specific user and can depend only on capacity.
- Users can have different number of antennas



- $\Longrightarrow$ Code design for MIMO multicast is challenging.
- Goal: practical coding schemes for MIMO multicast.
- We show: challenges are successfully met via precoded integer-forcing (IF) combined with SIC.

MIMO Multicast Channel Model

- A transmitter equipped with M transmit antennas wishes to send the same message to K users.
- User i is equipped with N_i antennas.
- Channel matrix of user i: $[\mathbf{H}_i]_{N_i \times M}$.
- Set of channels: $\mathcal{H} = \{\mathbf{H}\}_{i=1}^{K}$.
- Received signal at user i

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x} + \mathbf{z}.$$

- Power constraint

$$\mathbb{E}(\mathbf{x}^H \mathbf{x}) \leq M \cdot \text{SNR}.$$

- Additive noise $\mathbf{z} \sim \mathcal{CSCN}(0, \mathbf{I})$.

Closed-Loop Multicast Capacity

- Closed-Loop: CSI is available at both transmission ends.
- The multicast capacity is attained by a Gaussian vector input, where the mutual information is maximized over all covariance matrices $\mathbf{Q}$ satisfying $\mathrm{Tr}(\mathbf{Q}) \leq M \cdot \mathrm{SNR}$

$$C(\mathrm{SNR}, \mathcal{H}) = \max_{\mathbf{Q}:\mathrm{Tr}(\mathbf{Q}) \leq M \cdot \mathrm{SNR}} \min_{\mathbf{H} \in \mathcal{H}} \log\det(\mathbf{I} + \mathbf{H}^H \mathbf{Q} \mathbf{H}).$$

- Figure of merit for assessing the performance of a scheme achieving rate $R(\mathrm{SNR}, \mathcal{H})$:

$$\textbf{Efficiency} = \eta(\mathrm{SNR}, \mathcal{H}) = \frac{\mathrm{R}(\mathrm{SNR}, \mathcal{H})}{\mathrm{C}(\mathrm{SNR}, \mathcal{H})}.$$

Known Practical Multicast Capacity Achieving Schemes

- There are several special cases where known linear modulation techniques achieve the multicast capacity when coupled with codes designed for a scalar AWGN channel:

Rx Ant	Tx Ant	User Num	SNR	Mod.
1	Any	2	Any	BeamForming
1	2	Any	Any	BF+Alamouti
Any	Any	Any	Very Low	BF+OSTBC
Any	Any	Any	Very High	NVD+IF
Any	Any	Moderate	Any	?

Precoded IF Equalization For Closed-Loop Multicast

- For SU Open-Loop, Ordentlich '13, et. al., considered:
  - Rx side - Integer Forcing Equalization
  - Tx side - Linear Precoding
- Showed that linear Non-Vanishing Determinant (NVD) precoder achieves the mutual information up to a constant gap for any channel.
- Open-loop scenario = limit of many users
  $\Longrightarrow$ results applicable also to closed-loop.
- However, the guaranteed gap to capacity is very large
  $\Longrightarrow$ not meaningful at moderate rates.
- We show that optimizing the precoder based on CSI, yields very good performance for closed-loop multicast for all SNR values.

Integer-Forcing Equalization: Review

- Consider the (SU) channel

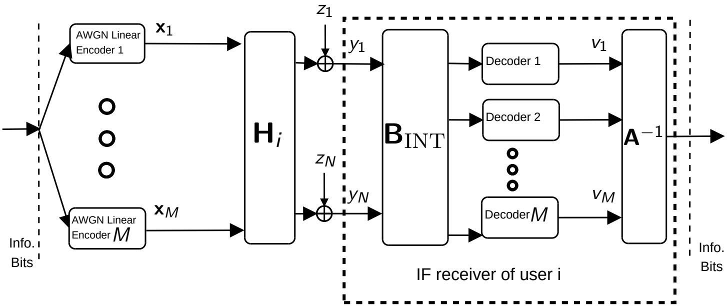
$$\mathbf{H} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

- At high SNR linear receiver front-end *inverts* the channel (ZF) thus resulting in *noise amplification*

$$\mathbf{H}^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}.$$

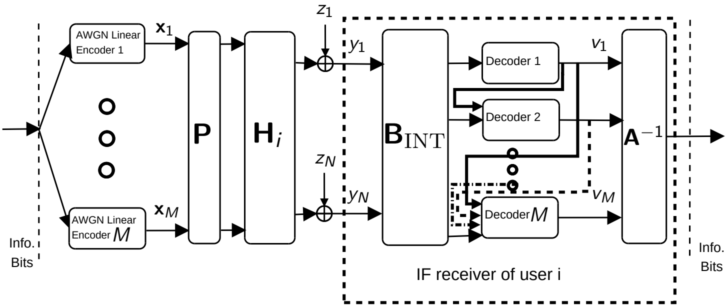
- Can we avoid noise amplification?
- IF idea: if all streams are coded with **same** linear code $\Longrightarrow$ signal at each antenna is a **valid codeword**.
- However, normal channels do not consist only of integers.
- Integer Forcing (IF) equalization generates an equivalent channel with integer entries.

Integer-Forcing Equalization: Review



- Information bits are fed into M encoders, each of which uses the same scalar *AWGN linear code*.
- Linear equalization matrix $\mathbf{B}_{\mathrm{INT}}$ is applied, such that the resulting equivalent channel $\mathbf{A} = \mathbf{B}_{\mathrm{INT}}\mathbf{H}$ is an integer matrix.
- In a practical implementation: computing $\mathbf{A}$ is done via LLL (polynomial complexity) once per code block.

Integer-Forcing Equalization: Review



- Integer matrix $\Longrightarrow$ the output of the channel (without noise) after applying a modulo operation is a valid codeword.
- We further consider a generalized version of the IF equalizer that incorporates **SIC** along with **linear precoding** at the transmitter.

IF-SIC Performance

- Using MMSE equalization the effective SNR at the m'th subchannel is $\mathsf{SNR}_{\mathrm{eff,m}} = \left(\mathbf{a}_m^T(\mathbf{I} + \mathsf{SNR}\mathbf{H}^H\mathbf{H})^{-1}\mathbf{a}_m\right)^{-1}$
- The effective SNR associated with the IF scheme is

$$\mathsf{SNR}_{\mathrm{eff}} = \min_{m=1,\ldots,M} \mathsf{SNR}_{\mathrm{eff,m}}$$

- Achievable rate without SIC (Zhan '10):
  $R_{\mathrm{IF}} < M \log(\mathsf{SNR}_{\mathrm{eff}})$
- Achievable rate with SIC (Ordentlich 13'):

$$R_{\mathrm{IF-SIC}} < M \max_{\mathbf{A}} \min_{m=1,\ldots,M} \log\left(\frac{\mathsf{SNR}}{\ell_{m,m}^2}\right)$$

where $\mathbf{L}$ is the following Choleskey decomposition

$$\mathbf{K}_{\mathbf{z}_{\mathrm{eff}}\mathbf{z}_{\mathrm{eff}}} = \mathbf{A}\left(\mathbf{I} + \mathsf{SNR}\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{A}^T = \mathbf{L}\mathbf{L}^T$$

Goal: Maximize Efficiency of Precoded IF-SIC

- With precoding, the resulting effective multicast channel is

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{P}\mathbf{x} + \mathbf{z} = \tilde{\mathbf{H}}_i\mathbf{x} + \mathbf{z}.$$

- For a given precoding matrix $\mathbf{P}$ achievable efficiency is known.
- Precoding matrix $\mathbf{P}$ can be written as $\mathbf{P} = \mathbf{Q}^{1/2}\mathbf{U}$ where $\mathbf{Q}^{1/2}$ is covariance shaping matrix and $\mathbf{U}$ is a unitary matrix.
- Effective channel: $\tilde{\mathbf{H}}_i = (\mathbf{H}_i\mathbf{Q}^{1/2})\mathbf{U}$.
- We take $\mathbf{Q}$ to be the capacity achieving covariance matrix.
  $\Longrightarrow$ Optimization reduces to find optimal $\mathbf{U}$.
- The achievable efficiency is given by

$$\frac{R_{\mathrm{P-IF-SIC}}(\mathrm{SNR}, \mathcal{H})}{C} = \frac{\max_{\mathbf{U}} \min_{\mathbf{H}\in\tilde{\mathcal{H}}} R_{\mathrm{IF-SIC}}(\mathrm{SNR}, \mathbf{H})}{C}.$$

Figure of Merit

- We assess the performance statistically where the set of channels $\mathcal{H}$ is viewed as drawn from an ensemble of channels.
- For a given scheme, we define the **outage efficiency** associated with the ensemble as

$$\eta_{x\%}(\mathrm{SNR}, M) = \max_{\Psi} \Pr(\eta(\mathrm{SNR}, M) < \Psi) = x\%$$

- For comparison purposes, we compare with the performance of several open-loop modulation methods:
  - OSTBC (which amounts to Alamouti modulation in case of two transmit antennas).
  - Perfect code (which amounts to golden code in case of two transmit antennas) coupled with the IF-SIC receiver.

Setup

- Both the transmitter and receiver have two antennas (2×2 MIMO channels).
- Two ensembles were considered:
  - **Rayleigh fading** - all matrix entries are circularly-symmetric complex normal random variables and are drawn independently of each other.
  - **Equal WI-MI with uniform distribution on singular values** - elements in this ensemble can be described as

$$\mathbf{H} = \mathbf{V}_1 \begin{bmatrix} \tilde{\sigma_1} & 0 \\ 0 & \tilde{\sigma_2} \end{bmatrix} \mathbf{V}_2.$$

- For each channel drawn from the ensemble we searched numerically for optimal $\mathbf{U}$.

Numerical Results - Rayleigh Fading

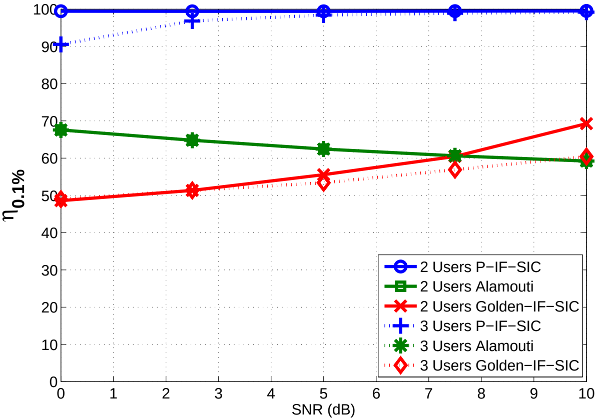


Figure : Outage efficiency for 2×2 channels drawn from ensemble I with outage probability of 0.001.

Numerical Results - Equal WI-MI

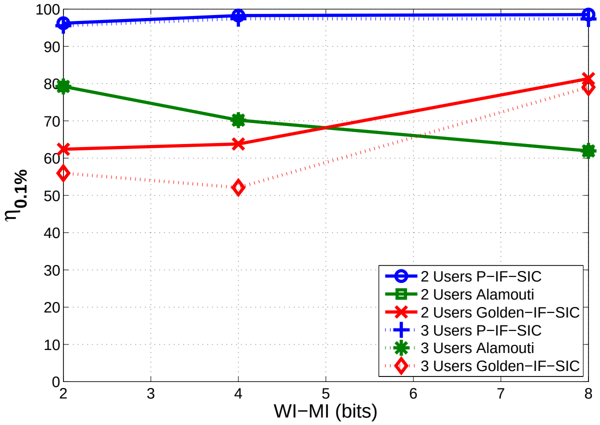


Figure : Outage efficiency for 2×2 channels drawn from ensemble II with outage probability of 0.001.

Efficiency As a Function of The Number Of Users

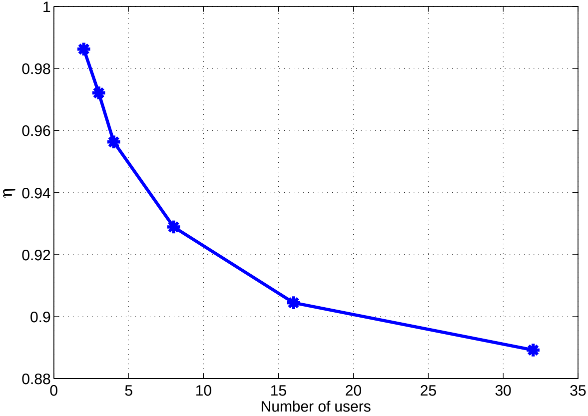


Figure : Channels are 2×2 matrices drawn from ensemble II with outage probability of 0.001, WI-MI=4

Results With Scalar Modulo

- Thus far: assumed multi-dimensional (optimal) modulo.
- Using scalar modulo results in a loss of up to 0.254 bits per real dimension.
- Maybe worth the effort to implement high-D modulo!

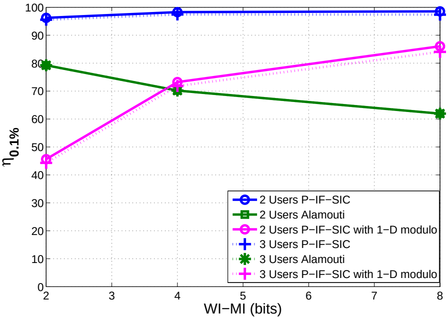


Figure : Outage efficiency for 2×2 channels drawn from ensemble II with outage probability of 0.001, where a 1-D modulo operation is performed.